MATHEMATICAL THEORY OF EVIDENCE TO DENGUE FEVER DETECTION

Andino Maseleno, Fauzi, Muhammad Muslihudin

Abstract

This paper presents Dempster-Shafer Theory for dengue fever detection. Sustainable elimination of dengue fever as a public-health problem is feasible and requires continuous efforts and innovative approaches. In this research, we used Dempster-Shafer theory for detecting dengue fever diseases and displaying the result of detection process. The Dempster-Shafer theory is a mathematical theory of evidence. Dengue fever diseases have the same symptoms with babesiosis, lyme, malaria, and west nile. We describe six symptoms as major symptoms which include fever, red urine, skin rash, paralysis, headache, and arthritis. Dempster-Shafer theory to quantify the degree of belief, our approach uses Dempster-Shafer theory to combine beliefs under conditions of uncertainty and ignorance, and allows quantitative measurement of the belief and plausibility in our identification result.

Keywords: Dengue Fever Diseases, Detection, Dempster-Shafer Theory

1. INTRODUCTION

There are many dengue fevers that are the primary or intermediate hosts or carriers of human diseases. Mosquitoes are perhaps the best known invertebrate vector and transmit a wide range of tropical diseases including malaria, dengue fever and yellow fever. Malaria is the world’s most devastating disease and kills more people than any other communicable disease except Tuberculosis. Between 2000 and 2009, out of 36 countries listed as endemic, 24 received the exclusive support of WHO (World Health Organization) either to assess the epidemiological status of HAT (human African trypanosomiasis) or to establish control and surveillance activities [1]. Another large group of vectors are flies. Sandfly species transmit the disease leishmaniasis, by acting as vectors for protozoan Leishmania species, and tsetse flies transmit protozoan trypanosomes (Trypanosoma brucei gambiense and Trypanosoma brucei rhodesiense) which cause African Trypanosomiasis (sleeping sickness). Ticks and lice form another large group of invertebrate vectors. The bacterium Borrelia burgdorferi, which causes Lyme Disease, is transmitted by ticks and members of the bacterial genus Rickettsia are transmitted by lice. For example, the human body louse transmits the bacterium Rickettsia prowazekii which causes epidemic typhus. Some systems for diagnosis in dengue fever diseases have been developed which were expert system for identifies forest dengue fevers and proposes relevant treatment [2], and expert system of diseases and dengue fevers of jujube based on neural networks [3]. Actually, according to researchers knowledge, Dempster-Shafer theory of evidence has never been used for built a system for detecting dengue fever diseases.

2. MATHEMATICAL THEORY OF EVIDENCE

The Dempster-Shafer (D-S) theory [4] or the theory of belief functions is a mathematical theory of evidence which can be interpreted as a generalization of probability theory in which the elements of the sample space to which nonzero probability mass is attributed are not single points but sets [5]. The sets that get nonzero mass are called focal elements. The sum of these probability masses is one, however, the basic difference between D-S theory and traditional probability theory is that the focal elements of a Dempster-Shafer structure may overlap one another. The D-S theory also provides methods to represent and combine weights of evidence.

m: 2^Θ → [0,1] is called a basic probability assignment (bpa) over Θ if it satisfies

m (Ø) = 0 and

\[ \sum_{S \in \Theta} m (S) = 1 \]
From the basic probability assignment, the upper and lower bounds of an interval can be defined. This
interval contains the precise probability of a set of interest and is bounded by two nonadditive continuous
measures called Belief (Bel) and
Plausibility (Pl). The lower bound for a set \( A \), \( Bel(A) \) is defined as the sum of all the basic probability
assignments of the proper subsets \( (B) \) of the set of interest \( (A) \) \( (B \subseteq A) \). Formally, for all sets \( A \) that are
elements of the power set, \( A \in 2^\Theta \)

\[
\sum_{B \subseteq A} m(B) = 1
\]

(2)

A function \( PL: 2^\Theta \to [0,1] \) is called a plausibility function satisfying

\[
\sum_{B \cap A \neq \emptyset} m(B) = 1
\]

(3)

The plausibility represents the upper bound for a set \( A \), and is the sum of all the basic probability
assignments of the sets \( (B) \) that intersect the set of interest \( (A) \) \( (B \cap A \neq \emptyset) \). The precise probability \( P(A) \) of
an event (in the classical sense) lies within the lower and upper bounds of Belief and Plausibility, respectively:

\[
Bel(A) \leq P(A) \leq PL(A)
\]

(4)

In this research, we used Dempster-Shafer theory for detecting dengue fever diseases and displaying the
result of detection process. Dengue fever diseases detection used Dempster-Shafer theory for decision
support process. Flowchart of dengue fever diseases detection shown in Figure 1.

Dengue fever diseases which include babesiosis, dengue fever, lyme, malaria, and west nile. We describe
six symptoms as major symptoms which include fever, red urine, skin rash, paralysis, headache, and
arthritis. Basic probability assignment for each symptoms can be seen in Table 1.

<table>
<thead>
<tr>
<th>SYMPTOM</th>
<th>DISEASE</th>
<th>BASIC PROBABILITY ASSIGNMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CONDITION 1</td>
</tr>
<tr>
<td>FEVER</td>
<td>BABESIOSIS</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>DENGUE FEVER</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MALARIA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>WEST NILE</td>
<td></td>
</tr>
<tr>
<td>RED URINE</td>
<td>BABESIOSIS</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Fig. 1 Flowchart of dengue fever diseases detection
3. DETECTING DENGUE FEVER DISEASES

The following will be shown detecting dengue fever diseases using Dempster-Shafer Theory. Dengue fever diseases which include babesiosis \(B\), dengue fever \(DF\), malaria \(M\), west nile \(WN\), and lyme \(L\).

1. Symptom 1: Fever
Fever is a symptom of babesiosis, dengue fever, lyme, malaria, and west nile with a bpa of 0.45, so that:
\[
m_1 \{ B, DF, M, WN \} = 0.45
\]
\[
m_1 \{ \Theta \} = 1 - 0.45 = 0.55
\]

2. Symptom 2: red urine
Red urine is a symptom of babesiosis with a bpa of 0.55, so that:
\[
m_2 \{ B \} = 0.55
\]
\[
m_2 \{ \Theta \} = 1 - 0.55 = 0.45
\]
With red urine symptom then required to calculate the new bpa values for some combinations \(m_3\).

**Combination rules for the \(m_3\) can be seen in the Table II.**

<table>
<thead>
<tr>
<th>(B)</th>
<th>0.45</th>
<th>(B)</th>
<th>0.55</th>
<th>(B)</th>
<th>0.45</th>
<th>(B)</th>
<th>0.65</th>
<th>(B)</th>
<th>0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B, DF, M, WN)</td>
<td>0.45</td>
<td>(B)</td>
<td>0.25</td>
<td>(B)</td>
<td>0.20</td>
<td>(B)</td>
<td>0.25</td>
<td>(B)</td>
<td>0.25</td>
</tr>
<tr>
<td>(\Theta)</td>
<td>0.55</td>
<td>(B)</td>
<td>0.30</td>
<td>(B)</td>
<td>0.26</td>
<td>(B)</td>
<td>0.26</td>
<td>(B)</td>
<td>0.26</td>
</tr>
</tbody>
</table>

\[
m_3 \{ B \} = \frac{0.25 + 0.30}{1 - 0} = 0.55
\]
\[
m_3 \{ B, DF, M, WN \} = \frac{0.20}{1 - 0} = 0.20
\]
\[
m_3 \{ \Theta \} = \frac{0.25}{1 - 0} = 0.25
\]

3. Symptom 3: Skin Rash
Skin rash is a symptom of lyme with a bpa of 0.45, so that:
\[
m_4 \{ L \} = 0.45
\]
\[
m_4 \{ \Theta \} = 1 - 0.45 = 0.55
\]
With skin rash symptom then required to calculate the new bpa values for some combinations \(m_5\).

**Combination rules for the \(m_5\) can be seen in the Table III.**

<table>
<thead>
<tr>
<th>(B)</th>
<th>0.45</th>
<th>(B)</th>
<th>0.55</th>
<th>(B)</th>
<th>0.45</th>
<th>(B)</th>
<th>0.65</th>
<th>(B)</th>
<th>0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B, DF, M, WN)</td>
<td>0.30</td>
<td>(B)</td>
<td>0.26</td>
<td>(B, DF, M, WN)</td>
<td>0.11</td>
<td>(B)</td>
<td>0.26</td>
<td>(B, DF, M, WN)</td>
<td>0.11</td>
</tr>
<tr>
<td>(\Theta)</td>
<td>0.25</td>
<td>(\Theta)</td>
<td>0.11</td>
<td>(\Theta)</td>
<td>0.14</td>
<td>(\Theta)</td>
<td>0.14</td>
<td>(\Theta)</td>
<td>0.14</td>
</tr>
</tbody>
</table>

\[
m_5 \{ B \} = \frac{0.30}{1 - (0.25 + 0.09)} = 0.45
\]
\[
m_5 \{ B, DF, M, WN \} = \frac{0.11}{1 - (0.25 + 0.09)} = 0.17
\]
\[
m_5 \{ L \} = \frac{0.17}{1 - (0.25 + 0.09)} = 0.17
\]
\[
m_5 \{ \Theta \} = \frac{0.45}{1 - (0.25 + 0.09)} = 0.21
\]

4. Symptom 4: paralysis
Paralysis is a symptom of lyme with a bpa of 0.45, so that:
\[ m_6 \{ L \} = 0.45 \]
\[ m_6 \{ \Theta \} = 1 - 0.45 = 0.55 \]
With paralysis symptom then required to calculate the new bpa values for some combinations (m7). Combination rules for the m7 can be seen in the Table IV.

**TABLE IV. COMBINATION OF SYMPTOM 1, SYMPTOM 2, SYMPTOM 3, AND SYMPTOM 4**

<table>
<thead>
<tr>
<th></th>
<th>0.45</th>
<th>0.55</th>
</tr>
</thead>
<tbody>
<tr>
<td>{B}</td>
<td>0.45</td>
<td>0.25</td>
</tr>
<tr>
<td>{B, DF, M, WN}</td>
<td>0.17</td>
<td>0.08</td>
</tr>
<tr>
<td>{L}</td>
<td>0.17</td>
<td>0.08</td>
</tr>
<tr>
<td>{M}</td>
<td>0.21</td>
<td>0.09</td>
</tr>
</tbody>
</table>

\[ m_7 \{ B \} = \frac{0.25}{1 - (0.20 + 0.08)} = 0.35 \]
\[ m_7 \{ B, DF, M, WN \} = \frac{0.09}{1 - (0.20 + 0.08)} = 0.12 \]
\[ m_7 \{ L \} = \frac{0.11}{1 - (0.20 + 0.08)} = 0.15 \]

5. Symptom 5: headache

Headache is a symptom of malaria with a bpa of 0.55, so that:
\[ m_8 \{ M \} = 0.55 \]
\[ m_8 \{ \Theta \} = 1 - 0.55 = 0.45 \]
With headache symptom then required to calculate the new bpa values for some combinations (m9). Combination rules for the m9 can be seen in the Table V.

**TABLE V. COMBINATION OF SYMPTOM 1, SYMPTOM 2, SYMPTOM 3, SYMPTOM 4, AND SYMPTOM 5**

<table>
<thead>
<tr>
<th></th>
<th>0.45</th>
<th>0.55</th>
</tr>
</thead>
<tbody>
<tr>
<td>{B}</td>
<td>0.35</td>
<td>\Theta</td>
</tr>
<tr>
<td>{B, DF, M, WN}</td>
<td>0.12</td>
<td>{M}</td>
</tr>
<tr>
<td>{L}</td>
<td>0.36</td>
<td>\Theta</td>
</tr>
<tr>
<td>{M}</td>
<td>0.15</td>
<td>{M}</td>
</tr>
</tbody>
</table>

\[ m_9 \{ B \} = \frac{0.16}{1 - (0.19 + 0.19)} = 0.26 \]
\[ m_9 \{ B, DF, M, WN \} = \frac{0.05}{1 - (0.19 + 0.19)} = 0.07 \]
\[ m_9 \{ L \} = \frac{0.16}{1 - (0.19 + 0.19)} = 0.26 \]
\[ m_9 \{ M \} = \frac{0.07 + 0.08}{1 - (0.19 + 0.19)} = 0.21 \]
\[ m_9 \{ \Theta \} = \frac{0.07}{1 - (0.19 + 0.19)} = 0.09 \]

6. Symptom 6: arthritis

Arthritis is a symptom of dengue fever with a bpa of 0.65, so that:
\[ m_{10} \{ DF \} = 0.65 \]
\[ m_{10} \{ \Theta \} = 1 - 0.65 = 0.35 \]

With bleeding around the bite symptom then required to calculate the new bpa values for some combinations (m11). Combination rules for the m11 can be seen in the Table VI.
TABLE VI. COMBINATION OF SYMPTOM 1, SYMPTOM 2, SYMPTOM 3, SYMPTOM 4, SYMPTOM 5, AND SYMPTOM

<table>
<thead>
<tr>
<th></th>
<th>0.65</th>
<th></th>
<th>0.35</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(B)</td>
<td>0.26</td>
<td>Ø</td>
<td>0.17</td>
<td>Ø</td>
</tr>
<tr>
<td>(B, DF, M, WN)</td>
<td>0.07</td>
<td>(DF)</td>
<td>0.04</td>
<td>(B, DF, M, WN)</td>
</tr>
<tr>
<td>(L)</td>
<td>0.26</td>
<td>Ø</td>
<td>0.17</td>
<td>Ø</td>
</tr>
<tr>
<td>(M)</td>
<td>0.21</td>
<td>Ø</td>
<td>0.14</td>
<td>Ø</td>
</tr>
<tr>
<td>Ø</td>
<td>0.09</td>
<td>(DF)</td>
<td>0.06</td>
<td>Ø</td>
</tr>
</tbody>
</table>

\[
m_{11}(B) = \frac{0.09}{1 - (0.17 + 0.17 + 0.14)} = 0.17, \quad m_{11}(B, DF, M, WN) = \frac{0.02}{1 - (0.17 + 0.17 + 0.14)} = 0.04
\]

\[
m_{11}(L) = \frac{0.09}{1 - (0.17 + 0.17 + 0.14)} = 0.17, \quad m_{11}(M) = \frac{0.07}{1 - (0.17 + 0.17 + 0.14)} = 0.13
\]

\[
m_{11}(DF) = \frac{0.04 + 0.06}{1 - (0.17 + 0.17 + 0.14)} = 0.19, \quad m_{11}(\Theta) = \frac{0.03}{1 - (0.17 + 0.17 + 0.14)} = 0.06
\]

4. RESULTS

Figure 2, 3, 4, 5, and 6 are shown graphic of detection from each condition.

![Figure 2](image)

**Fig 2. Condition 1**

Figure 2 shows the graphic of Condition 1, we get the highest basic probability assignment is dengue fever that is equal to 0.15 which shows from the last calculation of Dempster-Shafer on symptom 6 which means the possibility of a temporary diseases is dengue fever.
Fig 3. Condition 2

Figure 3 shows the graphic of Condition 2, we get the highest basic probability assignment is malaria that is equal to 0.19 which shows from the last calculation of Dempster-Shafer on symptom 6 which means the possibility of a temporary diseases with symptoms of fever, red urine, skin rash, paralysis, headache, and arthritis is malaria.

Fig 4. Condition 3

Figure 4 shows the graphic of Condition 3, we get the highest basic probability assignment is dengue fever that is equal to 0.25 which shows from the last calculation of Dempster-Shafer on symptom 6 which means the possibility of a temporary diseases with symptoms of fever, red urine, skin rash, paralysis, headache, and arthritis is dengue fever.

Fig 5. Condition 4

Figure 5 shows the graphic of Condition 4, we get the highest basic probability assignments are lyme and malaria that is equal to 0.21 which shows from the last calculation of Dempster-Shafer on symptom 6 which means the possibility of a temporary diseases with symptoms of fever, red urine, skin rash, paralysis, headache, and arthritis are lyme and malaria.
5. RESULT

In this paper we describe six symptoms as major symptoms which include fever, red urine, skin rash, paralysis, headache, and arthritis. The simplest possible method for using probabilities to quantify the uncertainty in a database is that of attaching a probability to every member of a relation, and to use these values to provide the probability that a particular value is the correct answer to a particular query. The knowledge is uncertain in the collection of basic events can be directly used to draw conclusions in simple cases, however, in many cases the various events associated with each other. Knowledge based is to draw conclusions, it is derived from uncertain knowledge. Reasoning under uncertainty that used some of mathematical expressions, gave them a different interpretation: each piece of evidence may support a subset containing several hypotheses. This is a generalization of the pure probabilistic framework in which every finding corresponds to a value of a variable. Identification of dengue fever diseases can be performed using Dempster-Shafer Theory.

REFERENCES


